

Dynamical Properties of Large N Reduced Model of Yang-Mills Theory

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We study the large N reduced model of D -dimensional Yang-Mills theory with special attention to the dynamical aspects related to the eigenvalues of the $N \times N$ matrices, which correspond to the space-time coordinates in the IIB matrix model. We define a quantity which represents the uncertainty of the space-time coordinates and show that it is of the same order as the extent of the space time, which means that the classical space-time picture is maximally broken. The absence of the SSB of the Lorentz invariance is also shown.

1. Introduction

Recently the large N reduced models [1] have been revived in the context of nonperturbative formulations of string theory [2,3]. The IIB matrix model [3–5], which is the reduced model of ten-dimensional supersymmetric Yang-Mills theory, is expected to be a nonperturbative formulation of superstring theory.

In this article, we study the large N reduced model of Yang-Mills theory, which we refer to as the “bosonic model” in what follows. It is nothing but the bosonic part of the IIB matrix model. The action is given by

$$S = -\frac{1}{4g^2} \text{Tr}([A_\mu, A_\nu]^2), \quad (1)$$

where A_μ ($\mu = 1, \dots, D$) are $N \times N$ traceless hermitian matrices.

We investigate the dynamical aspects of the model related to the eigenvalues of A_μ , which correspond to the space-time coordinates in the IIB matrix model. One of the most important quantities is the extent of the space time defined by $R = \sqrt{\langle \frac{1}{N} \text{Tr}(A^2) \rangle}$. We also define a quantity which represents the uncertainty of the space-time coordinates and an order parameter for the SSB of the Lorentz invariance. We determine these values for the bosonic model by Monte Carlo simulation.

2. The extent of the space time

For $D > 2$ with sufficiently large N , the integral of A_μ is convergent without any cutoff because of an attractive potential between the eigenvalues of A_μ [6]. This enables us to absorb g by rescaling $A'_\mu = A_\mu/\sqrt{g}$, which means that g is nothing but a scale parameter. The large N behavior of R is, therefore, parametrized as $\sqrt{g}N^\omega$, since R is proportional to \sqrt{g} on dimensional grounds. In the IIB matrix model, ω plays an important role to deduce the space-time dimension, which is to be determined dynamically from the eigenvalue distribution of A_μ [7]. We determine ω for the bosonic model.

The observables we measure are the following.

$$\begin{aligned} (a) \quad & \left\langle \frac{1}{N} \text{Tr}(A^2) \right\rangle \times \left(\sqrt{g}N^{1/4} \right)^{-2}, \\ (b) \quad & \left\langle \frac{1}{N} \text{Tr}((A_\mu A_\nu)^2) \right\rangle \times \left(\sqrt{g}N^{1/4} \right)^{-4}, \\ (c) \quad & \left\langle \frac{1}{N} \text{Tr}((A^2)^2) \right\rangle \times \left(\sqrt{g}N^{1/4} \right)^{-4}, \\ (d) \quad & \left\langle \frac{1}{N} \text{Tr}(F^2) \right\rangle \times \left(\sqrt{g}N^{1/4} \right)^{-4}, \end{aligned} \quad (2)$$

where $F_{\mu\nu} = i[A_\mu, A_\nu]$. Here, we have normalized the above quantities so that they are dimensionless, and hence they are independent of g . We note that (d) can be obtained analytically as

$(d) = D(1 - 1/N^2)$ [8].

In the Fig. 1, we show our results of Monte Carlo simulation for $D = 4$ with $N = 4, 8, 16, 32, 64, 128, 256$. One can see that $(a) \sim (d)$ are constant for $N \gtrsim 16$. In Fig. 2, we plot the data for $D = 10$ with $N = 2, 4, 8, 16, 32$. Here again we find that $(a) \sim (d)$ are constant for $N \gtrsim 16$.

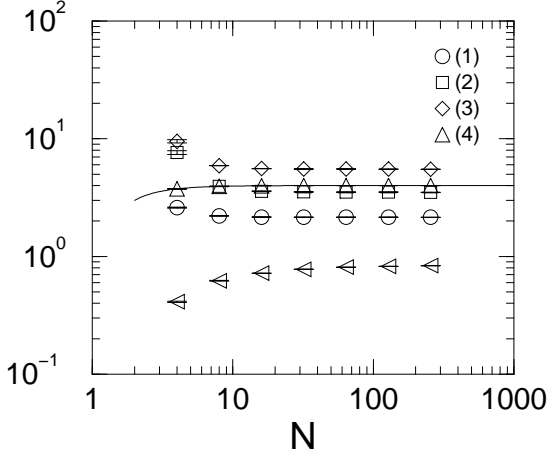


Figure 1. $(a) \sim (d)$ are plotted for $D = 4$. The solid line represents the exact result for (d) . The tilted triangles are the plot of $\langle \Delta^2 \rangle / (\sqrt{g} N^{1/4})^2$, which is discussed in Section 3.

From these results, we find that R is of the order of $O(\sqrt{g} N^{1/4})$, which means that the upper bound determined by the perturbative argument is saturated. The large N behaviors of $(a) \sim (c)$ are consistent with the predictions from the perturbative expansion and the $1/D$ expansion of this model [8].

3. Breakdown of the classical space-time picture

In the IIB matrix model, the eigenvalues of A_μ represent the space-time coordinates. If A_μ 's are commutable, they can be diagonalized simultaneously and the diagonal elements can be regarded

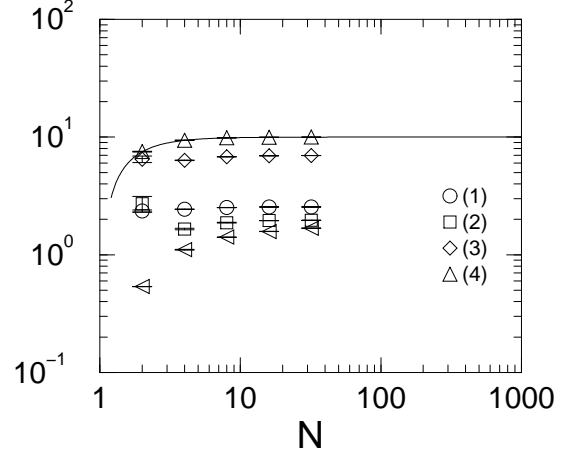


Figure 2. $(a) \sim (d)$ are plotted for $D = 10$. The solid line represents the exact result for (d) . The tilted triangles are the plot of $\langle \Delta^2 \rangle / (\sqrt{g} N^{1/4})^2$, which is discussed in Section 3.

as the classical space-time coordinates. Therefore it makes sense to ask to what extent the classical space-time picture is broken. To discuss this issue, we define a quantity which represents the uncertainty of the space-time coordinates.

We define such a quantity by the analogy to the quantum mechanics. We regard the matrices A_μ 's as the linear operators which act on the linear space, which we identify as the space of states of particles. We take an orthonormal basis $|e_i\rangle$ ($i = 1, 2, \dots, N$) and identify $|e_i\rangle$ with the state of the i -th particle. The space-time coordinate of the i -th particle is defined by $\langle e_i | A_\mu | e_i \rangle$. The uncertainty of the coordinate can be defined by

$$\delta(i)^2 = \sum_{\mu} \{ \langle e_i | A_\mu^2 | e_i \rangle - (\langle e_i | A_\mu | e_i \rangle)^2 \}. \quad (3)$$

We take an average of $\delta(i)^2$ over all the particles and define the gauge-invariant quantity by

$$\Delta^2 = \min_{\{|e_i\rangle\}} \left(\frac{1}{N} \sum_i \delta(i)^2 \right). \quad (4)$$

Note that $\Delta = 0$ if and only if A_μ are diagonaliz-

able simultaneously.

We plot $\langle \Delta^2 \rangle$ obtained by Monte Carlo simulation in Figs. 1 and 2. We find that $\langle \Delta^2 \rangle / (\sqrt{g} N^{1/4})^2$ tends to a constant for large N . Thus we conclude that $\sqrt{\langle \Delta^2 \rangle}$ is of the same order as R . If Δ were smaller than the typical distance between two particles, i.e., $\ell = RN^{-1/D}$, we might say that the classical space-time picture is good. Our result, however, shows that the classical space-time picture is maximally broken.

4. No SSB of Lorentz invariance

The SSB of Lorentz invariance is of paramount importance in the IIB matrix model, since if the space time is to be four-dimensional, the 10D Lorentz invariance of the model must be spontaneously broken.

The SSB of Lorentz invariance can be probed by

$$J = \frac{1}{D} I_{\mu\nu} I_{\mu\nu} - \left(\frac{1}{D} I_{\mu\mu} \right)^2, \quad (5)$$

where $I_{\mu\nu}$ is defined by $I_{\mu\nu} = \text{Tr}(A_\mu A_\nu)/N$. J represents nothing but the variation of the eigenvalues of $I_{\mu\nu}$.

If $\langle J \rangle / R^4$ is nonzero in the large N limit, the Lorentz invariance is spontaneously broken. Thus, $\langle J \rangle / (\sqrt{g} N^{1/4})^4$ can be considered as an order parameter of the SSB of Lorentz invariance. In Ref.[8], we show that when the large N factorization holds, the order parameter goes to zero in the $N \rightarrow \infty$ limit. Indeed, we show that at all orders of the $1/D$ expansion, the factorization holds and the order parameter behaves as $1/N^2$.

In Fig. 3 we plot $\langle J \rangle / (\sqrt{g} N^{1/4})^4$ against N for $D = 4, 6, 8, 10$. We see that the order parameter vanishes in the large N limit, which means that the Lorentz invariance is not spontaneously broken. Note also that the large N behavior of the order parameter can be nicely fitted to $1/N^2$. This is in accordance with the prediction by the $1/D$ expansion.

5. Summary

We studied the dynamical aspects of the reduced model of bosonic Yang-Mills theory. We

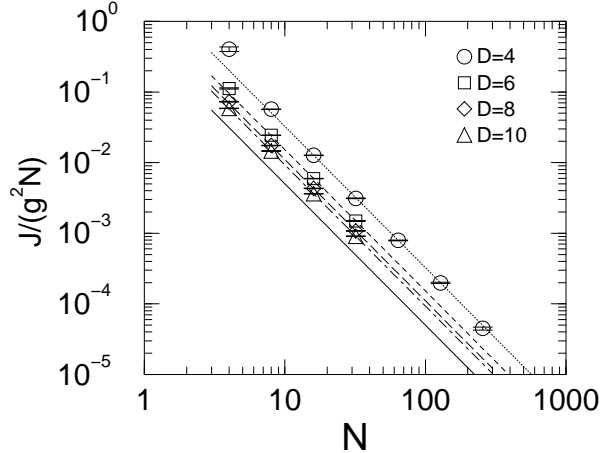


Figure 3. $\langle J \rangle / (\sqrt{g} N^{1/4})^4$ is plotted against N for $D = 4, 6, 8, 10$. The solid line represents the prediction for the $D \rightarrow \infty$ limit and the other lines represent the fit to the $1/N^2$ behavior [8].

determined the extent of the space time and showed that the classical space-time picture is maximally broken. We also showed that Lorentz invariance is not spontaneously broken. We expect that our findings for the bosonic model provide a helpful comparison when we investigate the IIB matrix model.

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